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Sustaining High Economic Growth in Sub-Saharan Africa: Knowledge and the Structure of the Economy

Yaw Nyarko*

New York University, USA

* Corresponding author: Yaw Nyarko. E-mail: yawnyarko1@gmail.com

Abstract

This paper discusses the role of knowledge and the structure of the economy in sustaining high economic growth, with an emphasis on Sub-Saharan Africa. A model of growth, involving learning from different activities, is proposed. Within the context of that model, a discussion is provided on some of the recent issues and debates around structural change of economies, and in particular on the measures for measuring the change. Special attention is placed on the recent measures such as Hausmann et al.'s (2007) EXPY and Lall's (1998) technology measures and the popular Herfindahl and Revealed Comparative Advantage measures used for analysing the change in export mixes.

JEL classification: O14, O21, O33, D90

1. Introduction

What causes and sustains high economic growth in Africa? The debate is still out on this question, and it is possible that there will never be a satisfactory answer. There are a few issues that seem clear just from the logic of the situation—governance, decent macro policies, etc. There are other issues too that should be addressed and discussed in part because they have received so much recent attention in the literature.

This paper discusses these issues of explaining growth from the standpoint of information and knowledge on the one hand, and structural change of the economy on the other hand. A theoretical growth model is provided and within that context, many of the current debates and metrics of change are discussed.

To begin, it should be remarked that most of the development debate in Sub-Saharan Africa over the past decade have been dominated by the United Nations Millennium Development Goals (MDGs). These specified a number of goals for nations to achieve, with the help of aid donors primarily from the West. The goals were principally in health and sanitation and also in primary education.

As the MDG process nears its end date, 2015, the debate has intensified as to what the development agenda for Africa should be. What should replace the MDGs as the dominant paradigm for development? The need for a different paradigm is based on the observation that although investments in health and primary education are important goals of social policy, it is unlikely that they will result in the high rates of growth needed to bring large numbers of Sub-Saharan Africans out of poverty. Furthermore, the huge reliance on foreign (primarily western) aid for both the MDGs and for national budget support is no way to achieve sustainable and high economic growth.

For these reasons, in this debate on new economic development paradigms, there have been many who have clamoured for a change in the basic structure of the economies of Africa. This clamouring is based on the observation that the structure of production of most Sub-Saharan African economies has not changed over the past 40 years or so, indeed over most of the post-independence era. The export mixes of most African countries have remained the same. The transit to manufacturing or higher value production has not taken place. There have been few cases of high-technology industries taking root. The goal of economic development and indeed the recipe for economic development, it has been argued, lies in changing the composition of output to higher value goods, to more technologically sophisticated goods.

Implicit in these arguments is the belief that sustained economic growth can be obtained by this change in the structure of the economies. But is this true? And even if it is true, what is the underlying economic theory that justifies the assertions that the change in the structure of the economy will result in economic growth and development. This paper takes a look at this question.

Economic revolutions have taken place in Africa, especially in the past. Many have documented, for example, the introduction of cocoa beans into Ghana by the local, TettehQuarshie, who smuggled some seeds from neighbouring Equatorial Guinea around the late 1800's. This resulted in Ghana having a major cocoa bean industry, at one time producing one-third of the World's output. This caused a major change in the economy of the area, as well of course major changes in social and political relationships. All of this has been well documented in the anthropological literature (see the classic Hill (1963)).

In modern times, many have spoken about Mauritius's ability to change the structure of its production from that of a sugar economy at independence in the mid 1960's to the creation of an economy, where some one-third of the GDP comes from high-end services, primarily upscale tourism. In terms of the promise of economic revolution for the future, many commentators have mentioned the potential of mobile technologies, and the mobile phone in particular, to lead to big changes in economic activity. Others have noted some new emerging industries—like flower exports to Europe from Kenya and Ethiopia as having the potential to be revolutionary.

So, how do we create new economic revolutions? Nyarko (2011a, b, c) has argued that markets and the incentives they create are fundamental to the creation of these economic revolutions. These in turn, it was argued, rely on the appropriate infrastructure—both economic and physical. Also of critical importance, it was argued in that piece, is the role of higher education, particularly at the tertiary level.

It appears as if, since independence, the ideas on development in Africa have gone full circle. Structural change, especially via import substitution industrialisation was in vogue right after independence of many Sub-Saharan African nations. This was followed by structural adjustment and the Washington consensus, getting the macro economy right. The MDGs then focused attention on poverty, and now, we are back to arguments on structural change—this time many are arguing for export-led change.

How should we advise the Government today on economic policy? How do we measure when governments are successful in their plans and goals? What are the metrics that should be used? The purpose of this paper is to discuss a few of the measures proposed in the literature for measuring economic change and to evaluate them within the context of a model of growth for which knowledge is critical in achieving the growth.

This paper is organised as follows. We begin with description of the structure of economies of Sub-Saharan Africa using metrics commonly used in the literature. Next, we focus on one particular metric, the EXPY measure of Hausmann *et al.* (2007). We then posit a theoretical growth model used previously by Jovanovic and Nyarko (1996) and again by Nyarko (2010). With this model, we discuss the EXPY framework and determine whether this is appropriate for analysing the growth prospects of nations. After making some brief comments on other measures used in the literature, we provide concluding remarks.

2. The structure of Sub-Saharan African economies

As mentioned in the introduction, one of the failures of the post-independence era is the lack of economic revolutions similar to those that occurred much earlier on with the advent of cash crops into Africa. We will now review quickly some of the measures of economic structure to illustrate the point that there has actually been little change in nature of economic production—in other words, not enough economic revolution.

2.1 Sector decomposition of domestic production

A key measure of the structure of the economy, and one for which statistics are often available, is the sector decomposition of GDP into Agriculture, Manufacturing, Services and Mining. We present some of the sector decomposition graphs below.

The analysis of sector decompositions and their changes has an extremely long history in the academic literature on economic growth. Lewis (1954), Fisher (1939), Clark (1940) and Baumol (1967) very early on wrote about how the changes in the sector decompositions of the economy are important for the long-run growth of economies.

Let us look at the data for Africa. There are data from the World Bank Development Indicators, which are fairly consistently available across African countries and go from 1970 through 2007. From these data, we can obtain for each year *t*, the percentage of GDP, which is attributable to agriculture, A_t, services, S_t, industrial manufacturing, I_t and mining activities, M_t, where $A_t + S_t, +I_t, +M_t = 1$.

One can then define change of sectors to be the change between the shares for current (or latest) year, 2007, and that of 1970. As these are vectors, a simple metric to evaluate the difference is the sum of squares:

$$\text{Metric} = (A_{2007} - A_{1970})^2 + (S_{2007} - S_{1970})^2 + (I_{2007} - I_{1970})^2 + (M_{2007} - M_{1970})^2.$$

Is this metric reasonable? Well, let us blindly apply it and see what it says for Africa over the 37-year period of the data. Indeed, let us rank the African nations and determine which has the greatest change under the metric above.

Table 1 orders the countries in terms of the value of the metric above. So, which African country has had the most economic change as defined by the metric above? Well, it is Equatorial Guinea (see Figure 1). The country has gone from sector shares in 1970 of $[A_t, S_t, I_t, M_t] = [0.63, 0.35, 0.01, 0.2]$ (after rounding) to sector shares in 2007 of $[A_t, S_t, I_t, M_t] = [0.04, 0.05, 0.05, 0.86]$. It is the mining of oil that has transformed the economy and so radically changed its metric. Congo, Chad, Angola and Botswana are all in the top 10 in terms of this metric, and for each, the cause is oil or in Botswana's case, diamonds. We present in Figure 2 the sector decomposition for Botswana as it is often studied as an African success story.

The other top performer in terms of our metric is Liberia. As can be seen in Figure 3, Liberia scores a high rank on the metric possibly for the wrong reasons. As its economy has been beset by war and civil unrest, the fraction of the economy in agriculture has increased tremendously. The economy has changed quite a bit, but maybe in the wrong direction.

For an economy that has had a relatively robust change for probably the right reasons, we can look at Mauritius, in Figure 4. It has seen the percentage of the economy in services to rise from 56% of the economy in 1970 to 74% of the economy. This has occurred because of a spectacular transformation into tourism and out of sugar production. For cases of very little change according to the above metric, one could look at Ghana or Tanzania shown in Figures 5 and 6.

In the search for the ideal metric for the use of sector decomposition, we see that our simple and naïve metric above has some pluses and some minuses. It picks up the relative success of transformation of Mauritius and the relative lack of change of Ghana and Tanzania. On the other hand, although Equatorial Guinea and other mineral-rich countries have changed by our metric, one may want a way of backing out the influence of oil and diamond prices in our measurements.

To deal with these problems there may be, for each country, an ideal sector decomposition $[A^*, S^*, I^*, M^*]$ that a country must aim for, and

Table 1: Sector Decomposition Transformations 1970–2007

Countries	Rank of Metric	Transformation Metric
Equatorial Guinea	1	1.07
Liberia	2	0.66
Congo	3	0.65
Chad	4	0.52
Mali	5	0.50
Malawi	6	0.47
Angola	7	0.46
Botswana	8	0.45
Zambia	9	0.40
Burundi	10	0.39
Nigeria	11	0.37
Congo (DRC)	12	0.37
Algeria	13	0.35
Uganda	14	0.33
Swaziland	15	0.31
Gambia	16	0.31
Namibia	17	0.29
Rwanda	18	0.28
Mauritius	19	0.26
Central African Republic	20	0.25
Ethiopia	21	0.25
Lesotho	22	0.24
Gabon	23	0.21
Mauritania	24	0.20
Comoros	25	0.20
Sierra Leone	26	0.19
Libyan Arab Jamahiriya	27	0.18
Kenya	28	0.17
Egypt	29	0.16
Niger	30	0.13
South Africa	31	0.13
Burkina Faso	32	0.12
Cameroon	33	0.12
Mozambique	34	0.12
Sao Tome and Principe	35	0.11
Seychelles	36	0.11
Guinea	37	0.11
Eritrea	38	0.11
Tunisia	39	0.11
Sudan	40	0.10
Madagascar	41	0.10
Ivory Coast	42	0.10

(*continued on next page*)

Table 1: *Continued*

Countries	Rank of Metric	Transformation Metric
Morocco	43	0.09
Zimbabwe	44	0.09
Ghana	45	0.09
Senegal	46	0.08
Togo	47	0.07
Tanzania, United Republic of	48	0.07
Cape Verde	49	0.06
Djibouti	50	0.06
Somalia	51	0.05
Benin	52	0.03

the metric of change evaluated could be relative to this ideal sector. The big question then would be what constitutes the ideal decomposition. Is it [0, 0, 1, 0] so that all activities are in industry? Or is it [0, 1, 0, 0], where all activities are in the service sector (the pure knowledge economy)? Unless a stand is made on what is the ideal state, it is hard to use any measure to calibrate transformation. On the other hand, once theory has given us for a particular country the ideal sector decomposition, it becomes extremely easy to design metrics to measure progress to the ideal state. If we have an 'ideal' decomposition, then we can evaluate the economy at any date t via the metric

$$\text{Metric} = (A_t - A^*)^2 + (S_t - S^*)^2 + (I_t - I^*)^2 + (M_t - M^*)^2. \quad (1)$$

2.2 Composition of exports

As a second indication of the structure of the economies, the lack of change has been in the exports. For many economies, exports are big relative to the total economy. One can ask whether nations have changed the export mix of their commodities from those that are considered low via some metric (e.g., technological sophistication) versus those that are considered much more superior. Again, there has been a huge volume of literature that talks about the product mix of exports and whether they are principally primary commodities, manufactured goods, etc. The United Nations agency UNCTAD has very good and frequently used data of the exports of nations (COMTRADE).

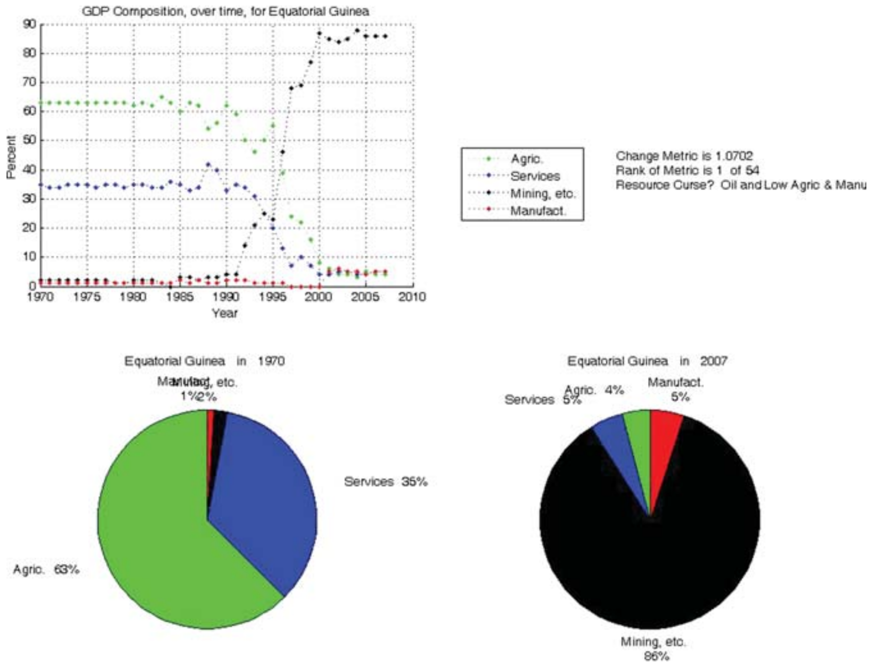

Figure 1: Sector Decomposition for Equatorial Guinea 1970–2007

In a collection of papers, Sanjaya Lall (Lall 1996, 1998, 2000) has explored various metrics for the sophistication of exports. These break exports down into categories such as primary goods, resource-based goods, manufactured products. Metrics are then based on the percentage of the export mix of nations in the various groups.

Hidalgo and Hausmann (2009) have a variant of this idea and have modelled a process where there is a ladder (or tree) showing how countries move up in the sophistication of their exports—with the need to pass through lower rungs of the ladder (or branches of the tree) to get to higher ones. Implicit in this theory, of course, are natural measures of change. The measures are computed via COMTRADE data and are based on the earlier EXPY measures by Hausmann *et al.* (2007). Because we will be commenting quite a bit on these models, it is important to spell them out a little bit more.

In that paper, the following notation is used. Let x_{jl} be the export of good l of country j. Then, $X_j = \sum_l x_{jl}$ the sum of all exports of country j. Let Y_j be the per capita GDP of Country j. Then, each good k is characterised by a

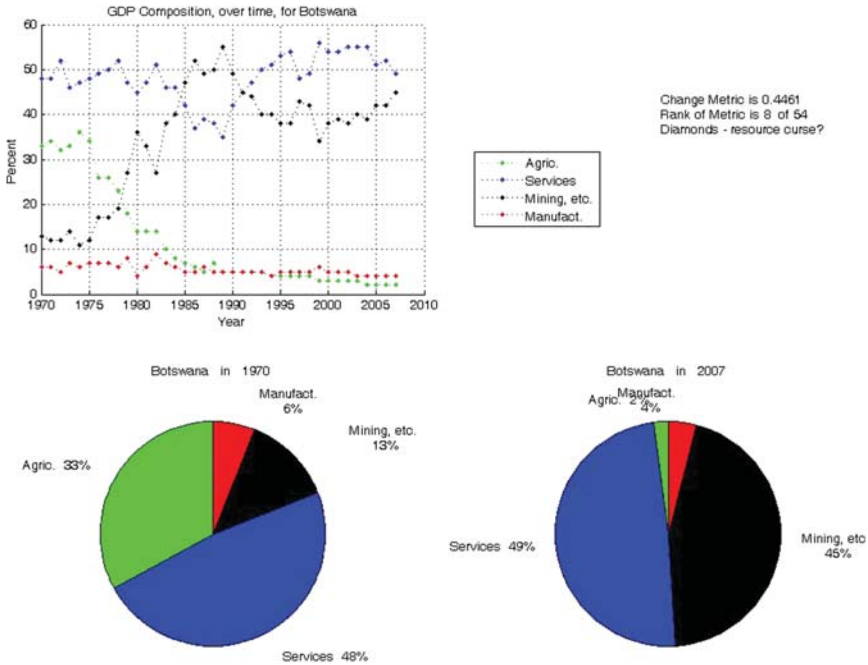

Figure 2: Sector Decomposition for Botswana 1970–2007

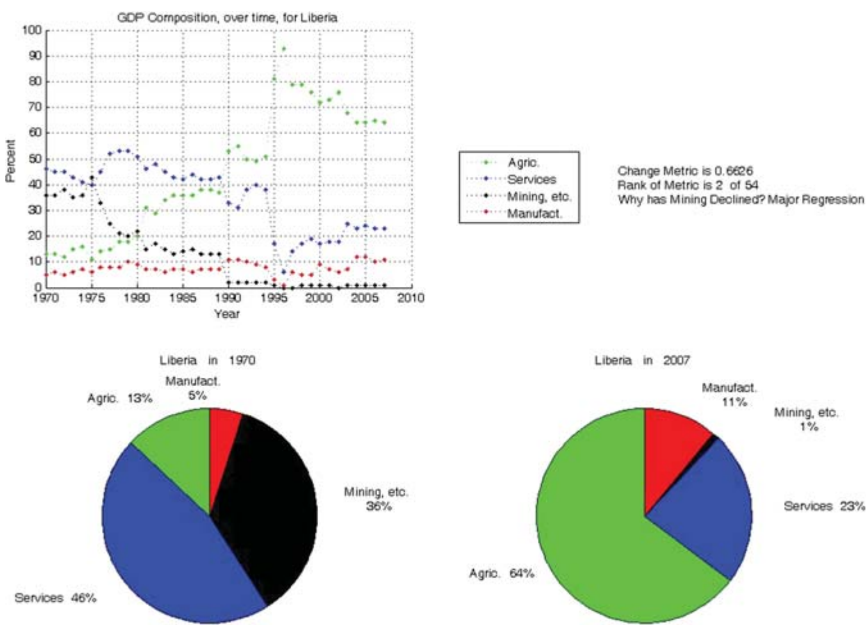

Figure 3: Sector Decomposition for Liberia 1970–2007

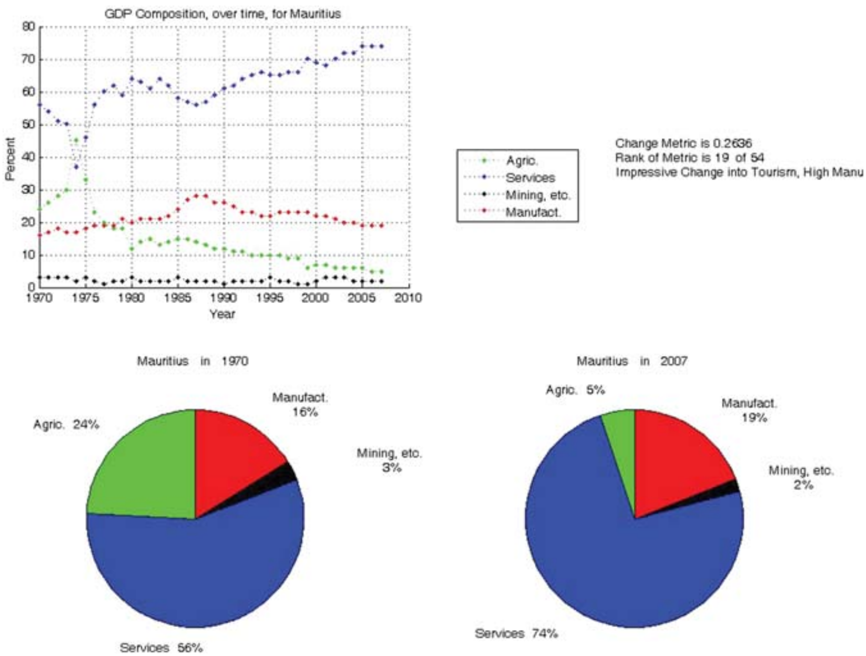

Figure 4: Sector Decomposition for Mauritius 1970–2007

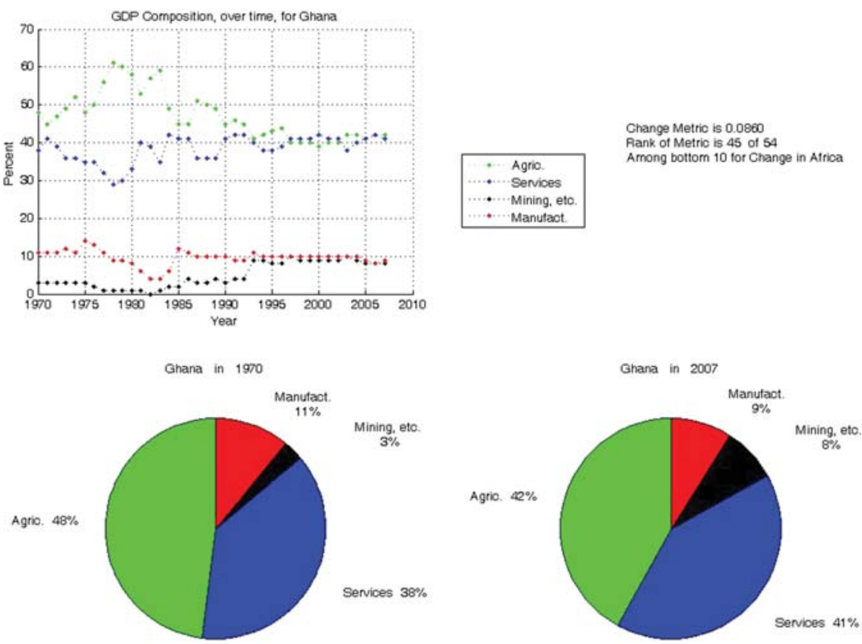

Figure 5: Sector Decomposition for Ghana 1970–2007

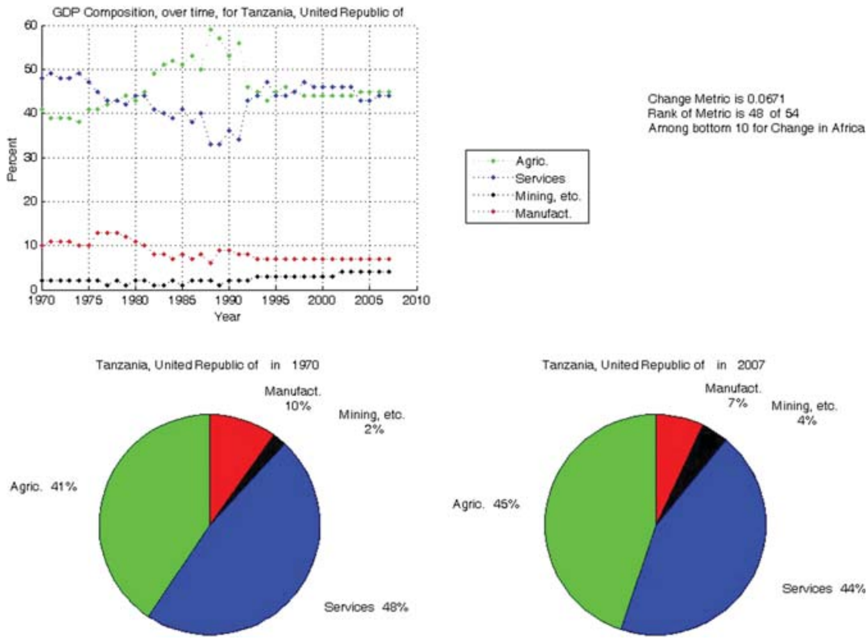

Figure 6: Sector Decomposition for Tanzania 1970–2007

'grade' or 'sophistication level' or productivity, denoted by PRODY_k, defined as follows:

$$\text{PRODY}_k = \sum_j \frac{(x_{jk}/X_j)}{\sum_j x_{jk}/X_j} Y_j. \tag{2}$$

Notice that PRODY_k is the weighted average of the GDP per capita levels of each country producing good j, Y_j where the weights are the export shares of good k in the basket of all exports of country j, normalised across all countries:

$$(x_{jk}/X_j)/\textstyle\sum_j (x_{jk}/X_j)$$

Each country is then evaluated according to the grades of the different commodities it has in its export basket. The income or productivity of a country's export basket, EXPY, is defined as weighted average of PRODY. The productivity of country I's export basket is, therefore, defined as

$$\text{EXPY}_i = \sum_j \left(\frac{x_{il}}{X_i}\right) \text{PRODY}_l. \tag{3}$$

The basic intuition behind the formulation above is that a good is deemed to be sophisticated if it is produced by richer nations. The higher the fraction of goods in a nation's export basket, which are produced by rich nations, the higher is the EXPY of that nation.

Hausmann *et al*. (2007) showed that the EXPY is highly correlated with per capita GDP and also growth rates. Some of the surprises they mention is that high-growth countries such as China and India have EXPY levels higher than prediction.

Furthermore, and intuitively, there is the question of whether the 'skill level' of a nation should be measured by the EXPY which is the average of the skill or sophistication levels of exports, or some measure of the upper bound of the range of goods produced.

By its very nature, the EXPY measure is intimately related to the GDP of a country. Indeed, suppose that you take a selection of countries whose exports are dominated by a few crops or minerals, and where in addition the countries are major producers of those commodities or are one of many with similar GDP per capita levels producing that commodity. Then, the EXPY measures for exports of those countries would be almost the same as the GDP per capita levels.

2.3 Other measures of change?

There are of course many other metrics used to discuss the sophistication of production or exports of countries. Some use measures like the percentage of exports that come from the top *N* goods. A high value of this indicates, as is the case of many African nations, a narrow range of exports and reliance on one or two cash crops or minerals. Others use measures of the concentration of exports, and the Herfindahl–Hirschman measure in particular. Finally, there is the Revealed Comparative Advantage metric that for each good measures the weight of that good in a country's export basic relative to the weight of the good in the basket of the total world exports.

2.4 Is there a link between growth rates, sustainability and structure of the economy?

Despite the fact that there is so much work done on the structure of the economy, the results on an empirical link between structure and growth are still not conclusive. Some papers show modest effect, whereas others claim such an effect can not be found. There is an extensive literature on this at this moment. We mentioned earlier the older papers by Lewis

(1954), Fisher (1939), Clark (1940) and Baumol (1967). The papers by Hausman, Rodrick and others mentioned earlier have all as an empirical goal showed the link between structure or sophistication of the economy and economic growth.

3. A theory of economic revolutions and moving up the quality ladder

There are many who have argued that economic development is the process of generating and implementing new ideas. Early economists like Hayek have articulated this in detail. Recently, Hausmann *et al.* (2007) have proposed economic development in terms of entrepreneurs searching for new activities. Nyarko (2010) has related this to some hypotheses of Diamond (1997).

How would one measure the success at transformation of a country or a sector? It will be a theory of productivity changes. It is the increased productivity which indicates that the country or sector is operating in a more sophisticated manner. One thinks of transforming agriculture from subsistence levels to modern versions. The productivity term will be essential.

3.1 Key elements of the model

We will provide below a model of growth based on human capital and learning to produce new goods.

The key ingredients of the model are as follows:

1. On a particular good, or grade of activity, there is learning by doing. However, sticking to that activity does not generate long-run growth.
2. Long-run growth only occurs by doing new things. But this requires human capital and production begins at low productivity levels.
3. There is, in addition, learning from other activities. Both Ghana and the USA produce chickens and beef. Yet in the USA, the production of chickens has benefited from the general technological sophistication of the economy outside the production of chicken. American chicken uses technologies that are more sophisticated because the general economy has more sophisticated technologies. In particular, all sectors are able to rise, in sync, as improvements in productivity in one sector of the economy move to others. These are the productivity spillovers mentioned earlier that are key to the sector change literature.

4 There is a role for policy intervention. An inferior activity in terms of current productivity may have spillovers via information on other sectors of the economy. In particular, there is a trade-off between productivity on an activity and the connectedness between that activity and the rest of the economy. Diversity, because of learning, is something that private firms do not internalise. There will, therefore, tend to be an undersupply of diversity in the economy. Government intervention or aid in a low productivity activity could, therefore, be justified from its role as a connector of others.

4. Applications to measures of change in the structure of the economy

The main use of the formal model below is to comment on the work on the structure of economies in relation to growth prospects. We do have below a growth model with predictions about how different activities could affect growth. The channel explored in the model relates to information about how to operate different activities. In that sense, it captures the notion of sophistication of production. It is a particular stylised view of the economy, where there are informational spillovers from one activity to another, so doing different activities happen to be important.

So, what insights does the model provide? First, as mentioned above, Hidalgo and Hausmann (2009) provide a model for how there are jumps from one activity to new ones. In their model, they provide an empirical explanation of why there can be a movement from one activity to another—one branch of a tree to another. The branches have to be close enough and similar enough. Our model provides some theoretical justification for this. The similarity in the branches is measured by how closely the economic activities are. This is a measure of correlation between the branches, which is captured in the work of Hidalgo and Hausmann (2009).

The model also shows that the diversity of production is not necessarily good in its own right. The new activity must be close enough to the core activities of nation in order for the learning on it to be of any use. Having a standalone enclave production activity provides few spillovers to the rest of the economy and, therefore, results in few growth benefits. Some have argued that aluminium smeltering plants working off hydroelectric dams is a case in point—Ghana's Volta River Aluminium company, VALCO being a prime example here.

5. The formal model

We now present a theoretical growth model that we believe captures some of the basic elements of the relationship between structure of the economy and change. This model is based on early work by Jovanovic and Nyarko (1996).

The model has a number of basic features. The first is that each good is characterised by a grade. At this time, the grade is exogenously specified and listed as $n = 1,2,3, \ldots$. In Hausmann *et al.* (2007), n is determined endogenously via the observation of what countries produce—in particular, higher n goods are those produced by richer nations.

The output given a grade n at any date t is given by $Y_t = q_t \cdot f(K, L, \ldots)$, where q_t represents the sophistication level of production on that grade, and K,L, ... represent the standard factors such as capital and labour that go into the standard production function f. As we are concentrating on growth, we shall focus exclusively on the q_t term and shut down the f channel by supposing that it is relatively unchanging and therefore enabling us to consider it fixed in our analyses of the growth dynamics. Instead, we will later on identify a different function, F that defines the dynamics of the productivity term q_t and is a function of human capital levels and knowledge in the economy.

We then model diversity or production of a range of goods as learning on different activities. This learning provides signals which enables the increase in the sophistication of production on all goods.

Growth occurs because on production, there is learning by doing that increases human capital and enables nations to upgrade the types of activities they are engaged in. Long-run growth only occurs through this upgrading of activities, similar to the intuition behind the EXPY and other models of structural change discussed earlier.

A key parameter in our model will be the relatedness of the different activities. A principal conclusion will be that the diversity of production in and of itself may not be good for economic growth. What is important is that the diverse activities complement each other and, therefore, enable each to pull the other up the quality ladder of production.

5.1 The one-good model

A technological line is made up of grades indexed by $n \in [0, \infty)$. The industry must choose a grade n to use and must also choose a decision variable z representing how to operate that grade, resulting in a net output in period t given by

$$q_t = \gamma^n[1 - (y_{nt} - z)^2], \quad \gamma > 1, \tag{4}$$

where

$$y_{nt} = \theta_n + w_{nt}. \tag{5}$$

Here y_{nt} is the target for the decision variable z, θ_n is the parameter relating to grade n; w_{nt} is a zero mean that is normal with zero mean and variance σ_w^2. The (representative firm in the) industry does not know the value of θ_n but has a prior which is normal with date t mean $E_t\theta_n$ and variance $\mathrm{var}_t(\theta_n)$, which we denote by $\mathbf{N}(E_t\theta_n, \mathrm{var}_t\theta_n)$. If grade n is chosen, the risk-neutral firm seeking to maximise expected net output would set

$$z_t = Ey_{nt} = E\theta_{nt} \tag{6}$$

resulting in the expected output

$$E_t q = \gamma^n[1 - \mathrm{var}_t(\theta_n) - \sigma_w^2]. \tag{7}$$

We will define the human capital s to be the reciprocal of the variance x, so that $s \equiv 1/x$; s is often referred to as the precision. After working on grade n, the industry will experience learning by doing that will increase its human capital. Specifically, the industry will observe y in (5) which, from Bayesian updating, results in a posterior human capital $\mathbf{H}_1(s)$ and variance $h_1(x)$given by

$$\mathbf{H}_1(s) = s + \frac{1}{\sigma_w^2} \quad \text{and} \quad h_1(x) = \frac{\sigma_w^2 x}{\sigma_w^2 + x}. \tag{8}$$

(For emphasis, note that for $s = 1/x$, we have $\mathbf{H}_1(s) = 1/h_1(x)$). We suppose that the parameters of any two grades n and $n + k$ for $k > 0$ are related by the following:

$$\theta_{n+k} = \alpha^{k/2}\theta_n + e_k, \tag{9}$$

where

$$e_k \sim N(0, \rho_k\sigma_\varepsilon^2) \quad \text{and} \quad \rho_k = \begin{cases} (1-\alpha^k)/(1-\alpha) & \text{for } \alpha \neq 1 \\ k & \text{for } \alpha = 1 \end{cases}. \tag{10}$$

The above is the generalisation of the AR-1 process to a diffusion process.

It will be useful to define the following two functions:

$$\mathbf{H}_2(s,k) = \frac{s}{\alpha^k + s\rho_k\sigma_\varepsilon^2} \quad \text{and} \quad h_2(x,k) = \alpha^k x + \rho_k\sigma_\varepsilon^2 \tag{11}$$

$$\mathbf{H}(s,k) = \mathbf{H}_1(\mathbf{H}_2(s,k)) \quad \text{and} \quad h(x,k) = h_1(h_2(x,k)). \tag{12}$$

If s is the human capital of grade n, then the function H_2 (s, k) (resp. $\mathbf{H}(s, k)$)gives the human capital on grade θ_{n+k} before (resp. after) learning by doing on grade $n + k$. Define s_k^{**} to be the fixed point of $\mathbf{H}(., k)$ (one can show that it exists and is unique, and that iterates of $\mathbf{H}(., k)$ from any s converge to s_k^{**}). In particular, if there is a jump of size k in each period followed by learning on that grade, the posterior human capital will converge to s_k^{**}We will define

$$\hat{x} = \frac{\sigma_\varepsilon^2}{1-\alpha} \quad \text{for } \alpha \neq 1 \text{ and } \hat{s} = \frac{1}{\hat{x}}. \tag{13}$$

Note that $\hat{s}$ is the value of s such that $\mathbf{H}(s, k) = s$ and will play a key role in our analysis below.

Define $G(x, k)$to be the expected net output when x is the variance on the current or status quo grade and a jump of size k is chosen as follows:

$$G(x,k) = \gamma^k[1 - \sigma_w^2 - \alpha^k x - \rho_k\sigma_\varepsilon^2]. \tag{14}$$

We write a production function F in terms of $s = 1\backslash x$ and the grade k as follows:

$$\mathrm{F}(s,k) = G\left(\frac{1}{s}\right). \tag{15}$$

The production function F is therefore a standard increasing and strictly concave in s.[1] We suppose that once a grade has been passed for a higher grade, it is never recalled.

Define $s^\star$ to be the value of s such that $F(s, 0) = F(s, 1)$.We now impose some conditions on our parameters, which we refer to as Assumption B, ensuring that here is long-run positive growth from all initial conditions. Jovanovic and Nyarko (1996) identified two cases. Case A was characterised by the conditions $\alpha\gamma < 1$; $F(\infty, 0) > F(\infty, 1)$ (or, equivalently, $G(0, 0) > G(0, 1)$) and $s^* < s_1^{**}$. In case A, initial human capital conditions affect long-run growth catastrophically—low human capital s (high x) industries have long-run growth, whereas high human capital s (low x) industries always use the same grade and hence never grow. In case B that we

[1] One feature of F is that for very low s (high x), the production function is negative. This feature will neither drive nor be important to the conclusions of this work.

use here but refer to as Assumption *B*, there is long-run positive growth from all initial conditions.

Assumption *B*: (i) $\alpha\gamma > 1$; (ii) for some $k > 0$, $0 < F(\infty, 0) < F(\infty, k)$ (or, equivalently, $G(0, 0) < G(0, k)$) and $x^* < x_1^{**}$; and (iii) if$\alpha < 1$, then $G(\hat{x}, 0) < 0$.

Parts (i) and (ii) of the above assumption were used by Jovanovic and Nyarko (1996) with $k = 1$ and deliver the conclusion that there are spurts of growth (upgrading) followed by spurts of only learning by doing on a given grade. Condition (iii) is a technical condition required to rule out the possibility that the optimal jump size is $+\infty$ from all human capital levels.[2]

Define $\kappa^*(s)$ to be the optimal jump size from human capital *s*. The lemma below indicates that from each *s*, there is a unique optimal jump size, and further that the jump size is increasing in the human capital *s*. This shows that at low values of the precision, lower upgrading is optimal. In comparing Hong Kong and Singapore, Young (1992) suggested that Singapore was upgrading at too high levels, suggesting that lower values of upgrading would have resulted in higher returns.

Proposition 1. *Suppose Assumption (B) holds. Then,* (i) $\kappa^*(s)$ *is uniquely defined and finite from each s and* (ii) κ^* *is increasing in s; in particular, for* $s > s'$, $\kappa^*(s) \geq \kappa^*(s')$, *with strict inequality if* $\kappa^*(s') > 0$. *More specifically,*

$$\kappa^*(s) = Max\{\bar{\kappa}^*(s), 0\}, \tag{16}$$

where

$$\bar{\kappa}^*(s) = \begin{cases} \frac{1}{\ln \alpha} \ln\left(\frac{-(\ln \gamma) G(\hat{x}, 0)}{\ln(\alpha\gamma)\left(\hat{x} - \frac{1}{s}\right)} \right) & \text{for} \alpha \neq 1 \\ \frac{(\ln \gamma) F(s, 0) - \sigma_\varepsilon^2}{\sigma_\varepsilon^2 \ln \gamma} & \text{for} \alpha = 1 \end{cases}. \tag{17}$$

Proof. of Proposition. This follows from simple calculus and some algebraic manipulations and is available in Nyarko (2010) or on request from the author.

Because the jump size determines the growth rate, we have a direct mapping between the growth rates and the human capital. The higher is the human capital, the higher is the jump in upgrading, so the higher is

[2] This follows from the observation that when $\alpha < 1$, $\lim_{k\to\infty} \frac{1}{\gamma^k} G(x, k) = 1 - \sigma_w^2 - \hat{x}$.

the growth of output. Increasing the knowledge (human capital) results in higher economic growth rates!

Before we proceed to the next section, it may be good to remark on some related literature here. Easterly and Levine's (1998) model is also similarly related to the set-up used here—of learning from neighbours in the next section. Hoff (1997) has also used a Bayesian model with learning by doing similar to that used here.

5.2 The correlation structure among goods

Now, suppose that we have a number of goods. Suppose that the upgrading by the representative firms in the industry for each good is as described in the earlier one-good model. The only difference between this section and the earlier one is in the learning by doing that now occurs not only from the own industry, but also from other industries. The information from others will be useful because the parameters of production—the θ's—of each nation are correlated.[3]

In particular, let Ψ denote both the set of goods and the number of goods. Each good has a technological line with different grades indexed by $n \in [0, \infty)$. The parameter of good $c \in \Psi$ is denoted by θ_n^c. At the beginning of each date, firms producing good c must choose a grade n^c and a decision variable z^c on the grade, resulting in an output q^c, as in equations (4) to (7).

Let

$$\bar{\boldsymbol{\theta}}_n = \{\theta_n^c\}_{c \in \Psi} \tag{18}$$

be the vector of grade $\underline{n}$ parameters of each good. We suppose that the elements of $\bar{\boldsymbol{\theta}}_n$ are correlated and, in particular, have a joint normal distribution with a variance-covariance matrix X that has the symmetric form

$$\begin{aligned} &X = xI(\rho_\theta) \\ &\quad \text{where } I(\rho_\theta) = \begin{bmatrix} 1 & \rho_\theta & \cdots & \rho_\theta \\ \rho_\theta & 1 & \cdots & \rho_\theta \\ \cdots & \cdots & \cdots & \cdots \\ \rho_\theta & \rho_\theta & \cdots & 1 \end{bmatrix}, \ x > 0 \text{ and } \rho_\theta \in [0, 1]. \end{aligned} \tag{19}$$

In particular, we suppose that there is a common variance $x > 0$ of each θ_n^c

[3] We assume that the learning by doing noise terms, $\{w_t^c\}$, are independent across goods. We will in later work take up the situation where the correlation is in the error terms of the learning by doing component.

and a common correlation coefficient ρ_θ between any θ_n^c and $\theta_n^{c'}$ for $c \neq c'$. Fix a date and let n^c be the grade operated by industry c at that date. After each industry has operated on its chosen grade, each industry will observe the vector of signals from all industries, its own and others, represented by the following vector:

$$Y_n = \{y_{n^c}^c\}_{c \in \Psi} \quad \text{where } y_{nt}^c = \theta_n^c + w_{nt}^c. \tag{20}$$

To get some explicit results, we will impose some symmetry conditions. Suppose that at the very first period, each good c has some mean and a common variance x on the parameter of its first grade, θ_1^c. Suppose further that the grade 1 parameters of any two parameters θ_1^c and $\theta_1^{c'}$ share a common correlation coefficient ρ_θ. Further, the industries will be assumed to have common values of all the other relevant parameters of the model $(\gamma, \sigma_\varepsilon^2, \sigma_w^2)$.

Then from the optimisation problem of the earlier section, it should be clear that each industry will choose the same optimal upgrading behaviour in the first period. Because they receive the same information, they will all update their beliefs and have identical values for the variance on the parameters of their own grades. They will then repeat the optimisation problem and again choose the same grades to operate on the next, and hence each subsequent period. Each good will be on the same grade at each date, under the above-mentioned conditions.

After observing the signals of all nations, each industry c will update its beliefs about its own parameter, θ_n^c. We let

$$X' = h_1(x, \Psi, \rho_\theta) \tag{21}$$

denote the updated or posterior variance covariance matrix after observation of the Ψ signals Y_n whose correlation structure is defined via (x, ρ_θ) in (19). A comparison with the one-good model indicates that $h_1(x)$ of the earlier section is the one-good version of $h_1(x, \Psi, \rho_\theta)$ of this section. The lemma below indicates that X' will have a form similar to (19). Furthermore, part (2) of the lemma has the key result we will use later—namely that the posterior variance is decreasing in the correlation coefficient ρ_θ between nations.

Lemma 2. (Posterior distribution)

1. The posterior variance covariance matrix is given by

$$h_1(x, \Psi, \rho_\theta) = x' I(\rho'_\theta), \tag{22}$$

where $x' > 0$ *and* $\rho'_\theta \in [0,\ 1]$,*and where, in particular,*

$$x' = \frac{\sigma_w^2 + x[1 + (\Psi - 2)\rho_\theta - \rho_\theta^2(\Psi - 1)]}{(\sigma_w^2/x) + (x/\sigma_w^2)[1 + (\Psi - 2)\rho_\theta - \rho_\theta^2(\Psi - 1)] + (\Psi - 2)\rho_\theta + 2} \tag{23}$$

and

$$\rho'_\theta = \frac{\rho_\theta \sigma_w^2}{\sigma_w^2 + x[1 + (\Psi - 2)\rho_\theta - \rho_\theta^2(\Psi - 1)]}. \tag{24}$$

1. Furthermore,

$$\frac{\partial \rho'_\theta}{\partial \Psi} < 0, \frac{\partial \rho'_\theta}{\partial \rho} > 0 \quad \text{and} \quad \frac{\partial x'}{\partial \Psi} < 0 \tag{25}$$

and

$$\frac{\partial x'}{\partial \rho} \text{is} \begin{cases} < 0 & \text{for } \rho > 0 \\ 0 & \text{for } \rho = 0 \end{cases}. \tag{26}$$

Proof. Follows from simple but tedious algebra and is available upon request.

If $s = 1\backslash x$ is the prior human capital, then the posterior human capital will be $s' = 1/x'$, where x' is given by the lemma above.

Remark 3. *As* ρ_θ *goes to 1, we should approximate the model with many signals on the same* θ*. This is easily seen to be the case because from (23) above we obtain*

$$\lim_{\rho_\theta \to 1} s' = s + \frac{\Psi}{\sigma_w^2} \tag{27}$$

with the right hand of the above being the posterior variance with Ψ independent signals on a common parameter θ_1.

Remark 4. *Equation (25) above also shows that as* Ψ *increases, the posterior variance decreases—this captures the fact that signals are better for information processing. Furthermore, as* Ψ *increases, the posterior correlation coefficient between any two different* θ*'s decreases.*

Remark 5. *It is interesting to note the limit of s'* as $\Psi \rightarrow \infty$:

$$\lim_{\Psi \rightarrow \infty} s' = \frac{1}{\sigma_w^2} + \frac{s}{1-\rho}. \tag{28}$$

As $\rho_\theta \rightarrow 1$, s' becomes the posterior human capital (precision) on a given θ after getting information on parameters that are almost perfectly correlated. As $\Psi \rightarrow \infty$, we would, therefore, expect the posterior human capital to go to ∞. The formula above shows this.

5.3 The optimal dynamics, growth paths

In the lemma of the subsection above, we stated conclusions in terms of the final human capital: in particular, we noted that the higher is the absolute value of the correlation coefficient between two industries, the higher is the human capital after information sharing. We restate this here as a proposition.

PROPOSITION 6. *The (common) posterior human capital is increasing in ρ_θ*

In the earlier sections, we showed that the human capital is directly related to the growth rate via the upgrading behaviour. Learning from other activities in the economy leads to increases in the human capital on each activity. The result above shows that the correlation between the different activities is of extreme importance. The human capital is higher, if there is greater correlation between the activities.

5.4 Implications of the model

The above section spelt out the details of a formal model of economic growth. Before the formal model was described, a narrative was provided on the application of the model to measures of change in the structure of the economy. Now that we have the formal model, it may be useful to recapitulate the main features of the model and the implications for the analysis of growth.

To repeat, a growth model was provided where learning takes place through (a) learning by doing on a particular activity, (b) learning from producing other goods and (c) upgrading the activities to increasingly more sophisticated goods. Production on one particular activity provides productivity gains ((a) above), but those gains ultimately die out. The only way to have sustained growth is by producing more sophisticated goods ((b) above). Each good helps the productivity of other goods ((c) above). This last particular feature means that having a wider range of goods being produced in the economy is good for growth. However, the benefit only accrues, if the two goods are related somehow to each other. In the formal model, this relationship was captured by the correlation between key parameters of the production functions of the two goods. Having a large number of goods is not too important, if the goods are not related to each other.

This provides an alternative view of the structure of the economy. Measures of structural change that merely measure the number of goods produced in the economy (e.g., number of goods, share of top *N* exports, Herfindahl), capture the element of diversity that is good, but do not capture the correlation between goods, which is bad. Measures of structural change like the EXPY notions mentioned earlier are good because they capture endogenously the relationship between goods, which is good. On the other hand, those models are usually not explicit in how or why the growth occurs by producing more goods that are sophisticated. The earlier section provided such a model. That is the principal purpose of this paper.

5.5 Shortcomings of the model

As with every model, there are, of course, a number of shortcomings that should be noted, upfront. Substantively, this paper does not yet provide a theory of which sector to invest in, although it provides a theory of how one can think through the problem.

On a technical note, the assumption that all goods in a country are on the same grade is unrealistic. This assumption was made to keep the model tractable to enable its key features to be quickly discerned.

6. Conclusion

This paper has taken on the question of modelling the structure of the economy and discovering how the structure of the economy results in the growth of the economy. A review of the classical definition of structure—sector shares—is provided. It is observed that there has been little change in the structure, by this metric, among African nations over the past 40 years or so. This paper then took a look at the other popular metrics of structural change: number of goods, share of top N exports, Herfindahl and particularly the EXPY concept of Hausmann *et al.* (2007).

In this context, a growth model was constructed which could in principle capture a number of the key features of Hausmann *et al.* (2007). In the context of the constructed model, the observation was made that the correlation between the different activities within the economy is very much of importance.

Future work will be devoted to digging deeper into the empirical implications of the theoretical model proposed.

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